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AP Physics C CNY Homework

1.Review and summarize key concept and formula we have learned in this semester. (Electricity and Magnetism part) (Write on A4 papers. One or two pages are enough.)

2.Preview Electric Circuits (Chapter 25 of Tipler's textbook)

3. Finish the preview problem assigned on College Board named by "winter vacation preview".

Note: The term 1 need to be submitted at the first class of the next semester.

目录

Coulomb's Law 库伦定律

		点电荷	平板电容
库伦力$\vec{F}$		$F=\frac{k\|q_1q_2\|}{r^2}$	
电场强度$\vec{E}$	$\vec{E}=\vec{F}/q_0\ (N/C)$	$\vec{E}=\frac{kq}{r^2}\hat{r}$	
电势V	$dV=-\vec{E}\cdot d\vec{l}$	$V=\frac{kq}{r}$	$V=-Ex$
电势能U	$dU=-\vec{F}\cdot d\vec{l}$	$U_e=q_0V$	$U=-Fx=q_0V$
考点：	● 电荷是分正负的，计算库伦力(的叠加)和电场强度(的叠加)需要根据电荷不同，注意方向。**Barron Ch11-3** ● 电荷是可以流动的。**Barron Ch11-1**		

<u>General steps for this kind of problems: $\vec{E},\vec{F},V$</u>

$$ds \Rightarrow dQ \Rightarrow d\vec{E} \leftarrow \text{Maybe } d\vec{E}_x$$

Small segment

Single variable

1D	
2D	$dA=2\pi RdR$ area of a washer
3D	$dV=4\pi R^2dR$ volume of a spherical shell

1D	λ	$dQ=\lambda d(length)$
2D	σ	$dQ=\sigma d(area)$
3D	ρ	$dQ=\rho d(volume)$

$d\vec{F}=k\frac{qdQ}{r^2}\vec{r}$	$\begin{cases} d\vec{E}_x=d\vec{E}\cos\theta \\ d\vec{E}_y=d\vec{E}\sin\theta \end{cases}$
$d\vec{E}=\frac{kdQ}{r^2}\vec{r}$	
$dV=\frac{kdQ}{r}$	

考点：	● 注意三个步骤，尤其最后一步$d\vec{E}$可能是只计算$d\vec{E}_x$或者 $d\vec{E}_y$ ● 电荷均匀分布的球体：半径 r 外部的电荷对半径 r 表面电场无影响。

Gauss's Law 高斯定律

<table>
<tr><td rowspan="2">Gauss's Law</td><td>电通量: Φ_{net}</td><td>$\Phi_{net} = \oint_s \vec{E} \cdot \vec{n} dA = \oint_s E_n dA$</td></tr>
<tr><td>当Φ均匀分布时</td><td>$\Phi_{net} = E_n A = \frac{Q_{in}}{\varepsilon_0}$</td></tr>
<tr><td>考点:</td><td colspan="2">● 通量的概念: 垂直于某个表面的量称为通量 ⇒ 电通量定义
● 均匀电场 ⇒ 高斯面上电通量均匀 ⇒ 高斯定律算通量或电场
● Φ_{net}: 穿过高斯表面的净 flux ⇒ 和此表面上的电荷注意区分
Q_{in}: 高斯面里面的净 charge</td></tr>
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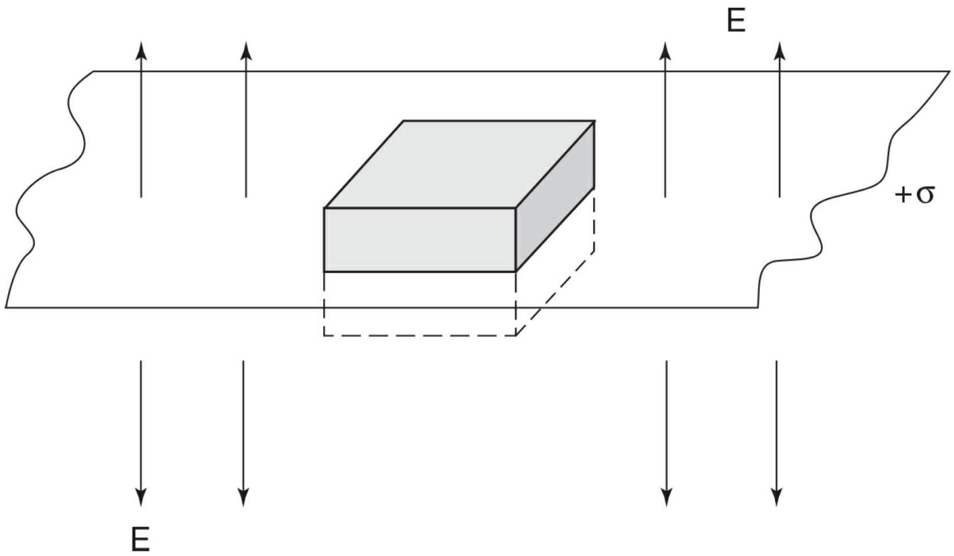
Example: (Barron P420)Calculate the electric field produced by an infinitely large sheet of charge with uniform charge density +σ C/m².



上下各一个 Gaussian surface: $A = x \times y$, 总面积 $2A$

The enclosed charge: $Q_{in} = \sigma A$

Gauss's Law: (当Φ均匀分布时) $\Phi_{net} = E_n A = \frac{Q_{in}}{\varepsilon_0}$

$$E_n(2A) = \frac{\sigma A}{\varepsilon_0} \Rightarrow E_n = \frac{\sigma}{2\varepsilon_0}$$

Electric Potential 电势

Potential energy 电势能

$$dU = -\vec{F} \cdot d\vec{l}$$
$$\vec{F} = q\vec{E}$$
$$dU = -q\vec{E} \cdot d\vec{l}$$

Definition of **Potential Difference 电势差**的定义

$$dV = \frac{dU}{q} = -\vec{E} \cdot d\vec{l}$$

For a finite displacement from point a to point b the change in potential is

$$\Delta V = V_b - V_a = \frac{\Delta U}{q} = -\int \vec{E} \cdot d\vec{l}$$

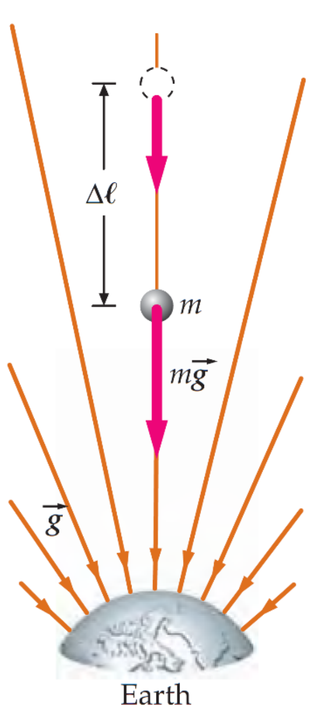
The unit Volt, $1V = 1J/C \Rightarrow 1J = 1V \cdot C = 1N \cdot m$

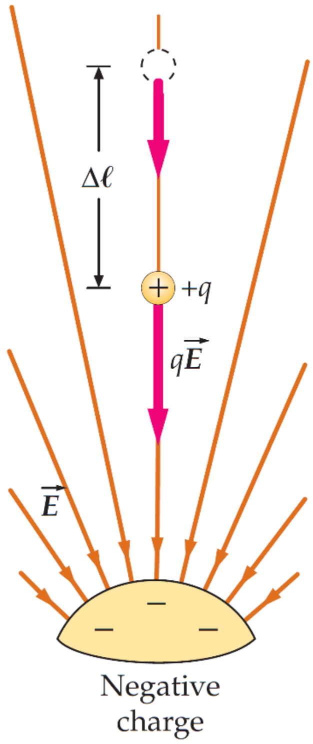
Please notice

1. The function V is called **electric potential 电势**
2. ΔV is call **electric potential difference 电势差(电压)**
3. V is a function of position – 在静电场中 V 只取决于位置
4. V is a scaler while $\vec{E}$ is a vector.
5. **Potential energy 电势能** $U = qV$
6. U and V are determined by choosing the same reference point zero.
7. The electric field is discontinuous by σ/ϵ_0 at points where there is a surface charge density σ. The potential is discontinuous at points occupied by a point charge or a line charge.

Potential and Electric Field 电场和重力场(p765)

Mass m	Charge q
Coulomb's Law $F = \frac{k\lvert q_1 q_2\rvert}{r^2}$ $k = 8.99 \times 10^9\ N \cdot m^2/C^2$	The University Gravitation $F_g = \frac{G m_1 m_2}{r^2}$ $G = 6.67 \times 10^{-11}\ N \cdot m^2/kg^2$
Electric Field $\vec{F} = q\vec{E}$ $E = \frac{kQ}{r^2}$	Gravitational Field $\vec{F} = m\vec{g}$ $g = \frac{GM_e}{r^2}$
Electric Potential Energy $U_e = VQ$	Gravitational Potential Energy $U_m = \mathrm{mgh}$
Potential Difference $V = \frac{U_e}{q} = -\vec{E} \cdot \vec{l}$	Height Difference $h = \frac{U_m}{mg}$





Capacitance 电容

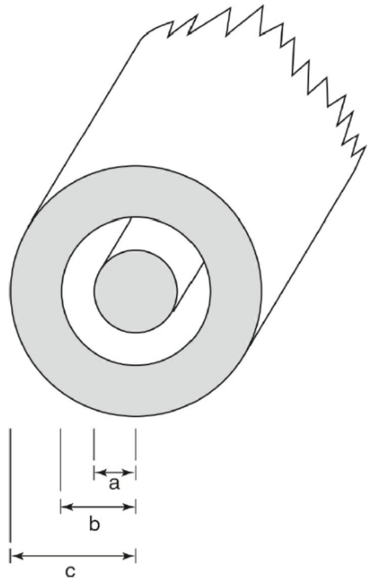
- Capacitance: $C = Q/V$
  Isolated spherical conductor $C = 4\pi\varepsilon_0 R$
  Parallel-plate capacitor $C = \varepsilon_0 A/d$
- Energy in Capacitors: $dU = VdQ = QdQ/C \Rightarrow U = \int \frac{Q}{C}dQ = \frac{Q^2}{2C}$

  $U = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2$
- Dielectrics: $C = kC_0 \quad \varepsilon = k\varepsilon_0$
  $\varepsilon_0 = 8.85 \times 10^{-12}\ F/m$
- Equivalent Capacitance:
  Parallel capacitors $C_{eq} = C_1 + C_2 + C_3 + \cdots$
  Series capacitors $1/C_{eq} = 1/C_1 + 1/C_2 + 1/C_3 + \cdots$

Calculating Capacitance Based on Geometry: $C = Q/\Delta V$

- 计算 E $\Leftarrow$ 当Φ均匀分布时 $\Phi_{net} = EA$
- 计算ΔV $\Leftarrow$ $dV = -\vec{E}\cdot d\vec{l}$
- 计算 C $\Leftarrow$ $C = Q/\Delta V$

Example: Consider the coaxial cable in the right figure, which contains a central cylindrical core of metal (radius a) surrounded by a cylindrical sheath of metal (inner radius b, outer radius c). Charge is moved from the sheath to the inner cylinder, so that charge density is then λ coulombs/length along the inner cylinder and −λ along the sheath.
What is the capacitance of a coaxial cable of length l (ignore edge effects, as in our treatment of a parallel plate capacitor).



Step 1: 计算 E

在 a<r<b 的位置，设置同轴高斯面

$$\Phi_{net} = \oint_s E_n dA \overset{E\perp A}{\Longrightarrow} \Phi_{net} = EA$$

$$\Phi_{net} = E(2\pi r\cdot l) = \frac{Q_{in}}{\varepsilon_0} = \frac{\lambda l}{\varepsilon_0}$$

$$E = \frac{\lambda}{2\pi r\varepsilon_0}$$

Step 2: 计算ΔV

$$V_b - V_a = \int_a^b dV = -\int_a^b Edr = -\int_a^b \frac{\lambda}{2\pi r\varepsilon_0}dr = \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{a}{b}$$

Step 3: C=Q/ΔV

$$\mathrm{C} = \frac{\mathrm{Q}}{\Delta \mathrm{V}} = \frac{\lambda l}{|V_b - V_a|} = \frac{2\pi\varepsilon_0 l}{\ln\frac{a}{b}}$$

Question 1

A narrow beam of protons produces a current of 1.6×10^{-3} A. There are 10^9 protons in each meter along the beam.

Of the following, which is the best estimate of the average speed of the protons in the beam?

(A) 10^{-15} m/s

(B) 10^{-12} m/s

(C) 10^{-7} m/s

(D) 10^{7} m/s

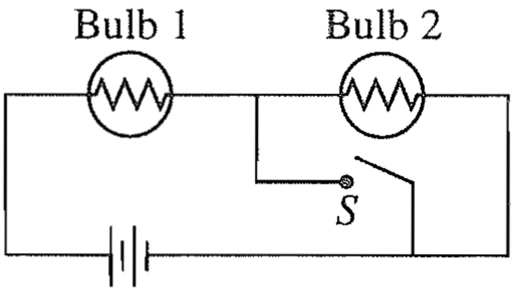
(E) 10^{12} m/s

$$I = \frac{\Delta Q}{\Delta t} = qnAv_d \Rightarrow v_d = \frac{I}{qnA} = \frac{1.6 \times 10^{-3}}{1 \times 10^9}$$

答案 (B)

Question 2

The circuit in the figure above contains two identical lightbulbs in series with a battery. At first both bulbs glow with equal brightness. When switch S is closed, which of the following occurs to the bulbs?



(A) Bulb 1: Goes out Bulb 2: Gets brighter

(B) Bulb 1: Gets brighter Bulb 2: Goes out

(C) Bulb 1: Gets brighter Bulb 2: Gets slight dimmer

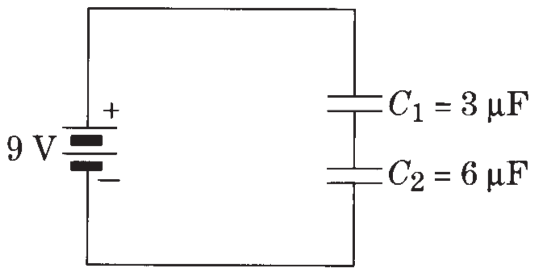
(D) Bulb 1: Gets slight dimmer Bulb 2: Gets brighter

(E) Bulb 1: Nothing Bulb 2: Gets out

答案 (B)

Question 3

Two capacitors initially uncharged are connected in series to a battery, as shown above. What is the charge on the top plate of C_1?



(A) –81 μC

(B) –18 μC

(C) –0 μC

(D) +18 μC

(E) +81 μC

Capacitors in series ⇒ same Q on C_1 and C_2

$C = \frac{Q}{V} \Rightarrow V = \frac{Q}{C}$ ⇒Q 相同，V 与 C 成反比

$\Rightarrow \frac{V_1}{V_2} = \frac{C_2}{C_1} \Rightarrow V_1 = 6\ (V)$

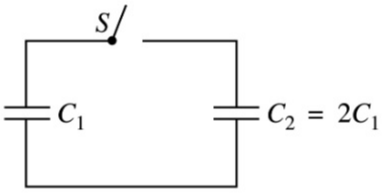
$\Rightarrow Q_1 = V_1 C_1 = 6 \times 3 = 18\ (\mu C)$

C_1 top plate is connected with the anode of the battery ⇒ +18 μC

答案 (D)

Question 4

A capacitor of capacitance C_1 is charged and then connected to another initially uncharged capacitor of capacitance $C_2 = 2C_1$, as shown above, with the switch S in the open position. When S is closed and the system comes to equilibrium, which of the following is true of the charges on the capacitors and the potential differences across them?



(A) Charge: $Q_1 = \frac{1}{2}Q_2$ Potential Difference: $V_1 = \frac{1}{2}V_2$

(B) Charge: $Q_1 = \frac{1}{2}Q_2$ Potential Difference: $V_1 = V_2$

(C) Charge: $Q_1 = Q_2$ Potential Difference: $V_1 = V_2$

(D) Charge: $Q_1 = Q_2$ Potential Difference: $V_1 = \frac{1}{2}V_2$

(E) Charge: $Q_1 = 2Q_2$ Potential Difference: $V_1 = V_2$

Capacitors in parallel ⇒ same V on C_1 and C_2

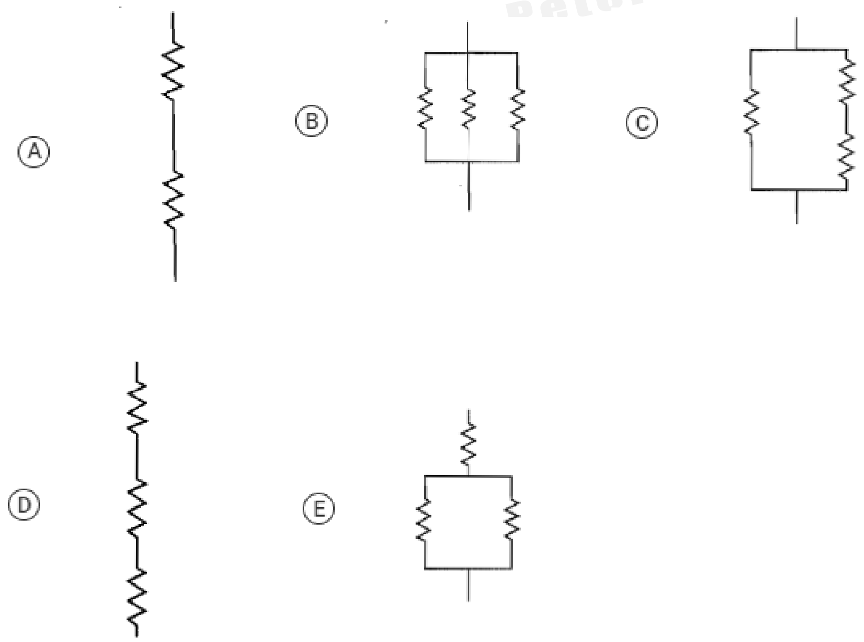
$C = \frac{Q}{V} \Rightarrow Q = CV$ ⇒V 相同，Q 与 C 成正比

$\Rightarrow Q_1 = \frac{1}{2}Q_2$

答案 (B)

Question 5

Which of the following combinations of 4 **Ω** resistors would dissipate 24 W when connected to a 12 V battery?



$P = IV = I^2R = \frac{V^2}{R} \Rightarrow R = \frac{V^2}{P} = \frac{12^2}{24} = 6\ (\Omega)$

答案 (E)

Question 6

Three $\frac{1}{2}\mu F$ capacitors are connected in series as shown in the diagram above.

The capacitance of the combination is

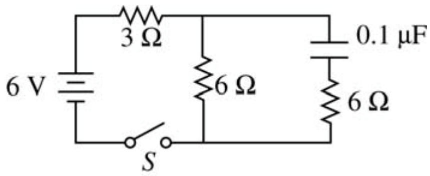
(A) $\frac{3}{2}\mu F$ (B) $1\mu F$ (C) $\frac{2}{3}\mu F$ (D) $\frac{1}{2}\mu F$ (E) $\frac{1}{6}\mu F$

Series capacitors $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$

答案 (E)

Question 7

In the circuit shown above, the capacitor is initially uncharged. The switch S is then closed.



Immediately after the switch is closed, the current in the 3 **Ω** resistor is most nearly

(A) 0A

(B) 0.20A

(C) 0.50A

(D) 0.67A

(E) 1.0A

Immediately after the switch is closed, $\begin{cases} 6\ \Omega \text{ circuit is connected} \\ 0.1\mu F + 6\ \Omega \text{ is open} \end{cases}$

⇒ 3 **Ω** and 6 **Ω** resistors share the 6V potential

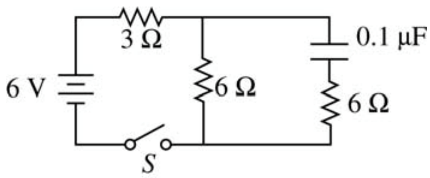
⇒ Voltage on 3 **Ω** resistor is (1/3) of 6V, that is 2V

$\Rightarrow I = \frac{V}{R} = \frac{2}{3}\ (A)$

答案 (D)

Question 8

In the circuit shown above, the capacitor is initially uncharged. The switch S is then closed.



A long time after the switch is closed, the current in the 3 **Ω** resistor is most nearly

(A) 0A

(B) 0.20A

(C) 0.50A

(D) 0.67A

(E) 1.0A

A long time after the switch is closed, $\begin{cases} 6\ \Omega \text{ circuit is connected} \\ 0.1\mu F + 6\ \Omega \text{ is open} \end{cases}$

⇒ 3 **Ω** and 6 **Ω** resistors share the 6V potential

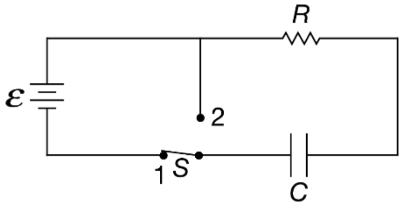
⇒ Voltage on 3 **Ω** resistor is (1/3) of 6V, that is 2V

$\Rightarrow I = \frac{V}{R} = \frac{2}{3}\ (A)$

答案 (D)

Question 9

In the circuit shown above, the switch S is initially in position 1. After capacitor C is fully charged, the switch is moved to position 2 at time t=0. Which of the following graphs best represents the current I as a function of time t in the resistor R?



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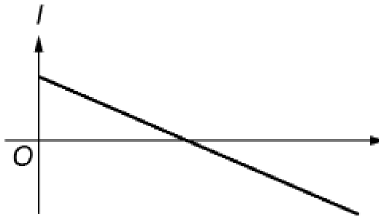

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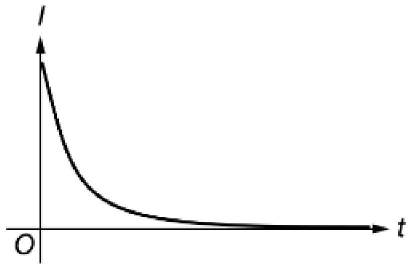

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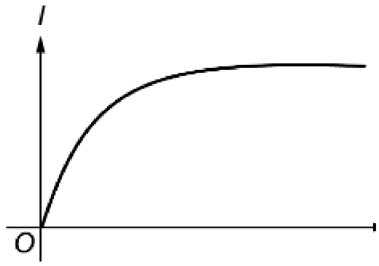

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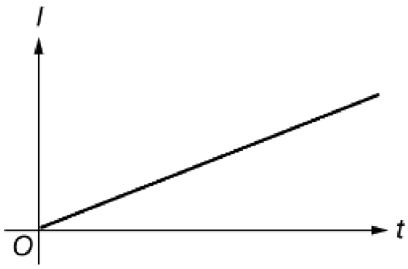

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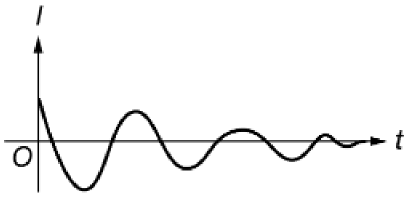
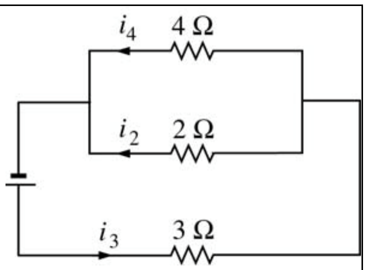


After the switch is moved to position 2, the capacitor C and resistor R setup a closed circuit and they are in series. The capacitor C will discharge and the resistor R (as well as the capacitor C) have the same discharging current of B.

答案 (B)

Question 10

Resistors of resistance 4 **Ω** , 2 **Ω** , and 3 **Ω** are connected in a circuit, as shown above. The currents through the resistors are i4 , i2 , and i3 , respectively. Which of these currents is least and which is greatest?

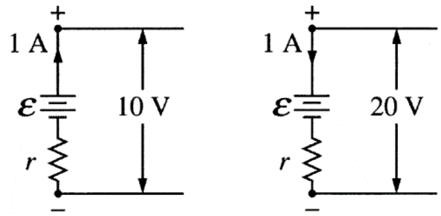


(A) Least i4 Greatest i3
(B) Least i2 Greatest i4
(C) Least i2 Greatest i3
(D) Least i3 Greatest i4
(E) Least i3 Greatest i2

i4 and i2 are in Parallel⇒ same Voltage. ⇒ I=V/R ⇒ i4 is the least one.
i3=i4+i2 ⇒ i3 is the greatest one.

答案 (A)

Question 11



The figures above show parts of two circuits, each containing a battery of emf ε and internal resistance r. The current in each battery is 1 A, but the direction of the current in one battery is opposite to that in the other. If the potential differences across the batteries' terminals are 10 V and 20 V as shown, what are the values of ε and r ?

(A) $\varepsilon=5\,V$ $r=15\Omega$

(B) $\varepsilon=10\,V$ $r=10\Omega$

(C) $\varepsilon=15\,V$ $r=5\Omega$

(D) $\varepsilon=20\,V$ $r=10\Omega$

(E) The values cannot be computed unless the complete circuits are shown.

In circuit 1, the battery is discharging. $\varepsilon=10V+(1A)*r$

In circuit 2, the battery is charging. $20V=\varepsilon+(1A)*r$

$\varepsilon=15V$ $r=5\Omega$

答案 (C)

Question 12

A resistor R and a capacitor C are connected in series to a battery of terminal voltage V_0. Which of the following equations relating the current I in the circuit and the charge Q on the capacitor describes this circuit?

(A) $V_0 + QC - I^2R = 0$

(B) $V_0 - \frac{Q}{C} - IR = 0$

(C) $V_0^2 + \frac{1}{2}\frac{Q^2}{C} - I^2R = 0$

(D) $V_0 - C\frac{dQ}{dt} - I^2R = 0$

(E) $\frac{Q}{C} - IR = 0$

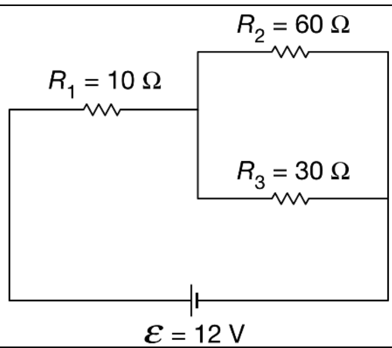
A resistor R and a capacitor C are connected in series ⇒ Total voltage = battery's terminal voltage.

Voltage of the resister: IR

Voltage of the capacitor: Q/C

答案 (B)

Question 13



In the circuit shown above, the potential difference across R_2 is most nearly

(A) zero

(B) 4.0 V

(C) 6.0 V

(D) 8.0 V

(E) 12 V

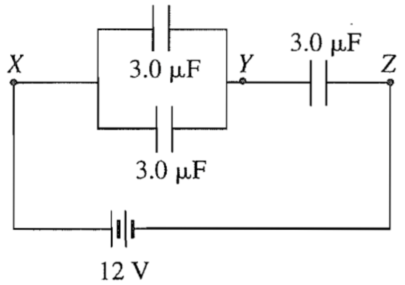
R_2 and R_3 are in parallel ⇒ Total resistance is $\frac{1}{R} = \frac{1}{R_2} + \frac{1}{R_3}$ ⇒ R=20 (Ω)

R and R_1 are in series $V(R_1) + V(R) = \varepsilon = 12\ (V)$ ⇒ $V(R) = 8\ (V) = V(R_2)$

答案 (D)

Question 14

Three identical capacitors, each of capacitance 3.0 μF, are connected in a circuit with a 12 V battery as shown above.



The potential difference between points Y and Z is

(A) zero

(B) 3 V

(C) 4 V

(D) 8 V

(E) 9 V

Capacitors in parallel ⇒ $C=C_1+C_2=6.0$ (μF)

Capacitors in series ⇒ same Q ⇒ $V = Q/C$ ⇒ V 与 C 成反比 ⇒ $V_{YZ} = 8.0\ (V)$

答案 (D)

Question 15

Wire of resistivity ρ and cross-sectional area A is formed into an equilateral triangle of side b, as shown above. The resistance between two vertices of the triangle, X and Y, is

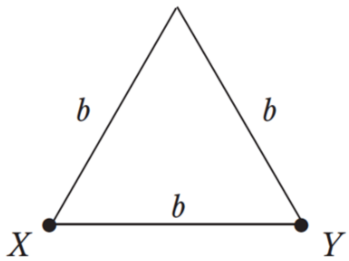
(A) $\frac{3}{2}\frac{A}{\rho b}$

(B) $3\frac{A}{\rho b}$

(C) $\frac{2}{3}\frac{\rho b}{A}$

(D) $\frac{3}{2}\frac{\rho b}{A}$

(E) $3\frac{\rho b}{A}$



Each resistor: $R = \frac{\rho b}{A}$

From X to Y, one circuit's resistance is 2R, the other one is R.

$$\frac{1}{R_{total}} = \frac{1}{R} + \frac{1}{2R} \Rightarrow R_{total} = \frac{2}{3}R = \frac{2}{3}\frac{\rho b}{A}$$

答案 (C)

Question 16

A metal wire has a resistance R when it is at a temperature T. The wire is melted and all of the metal is used to reform it into a new wire 4 times as long. What is the resistance of the new wire at temperature T ?

(A) R

(B) 2R

(C) 4R

(D) 8R

(E) 16R

For resistors: $R = \rho\frac{b}{A} \Rightarrow b_2 = 4b_1 \Rightarrow A_2 = \frac{1}{4}A_1 \Rightarrow R_2 = \rho\frac{4b_1}{\frac{1}{4}A_1} = 16R_1$

答案 (E)

Question 17

Three resistors having resistances of 3 Ω, 6 Ω, and 9 Ω, respectively, are connected in parallel with a 10 V battery. True statements about the circuit include which of the following?

I. The current in the 9 Ω resistor is three times the current in the 3 Ω resistor.

II. The potential difference across each resistor is the same.

III. The power dissipated in the 9 Ω resistor is greater than the power dissipated in either of the other two resistors.

(A) I only

(B) II only

(C) I and III only

(D) II and III only

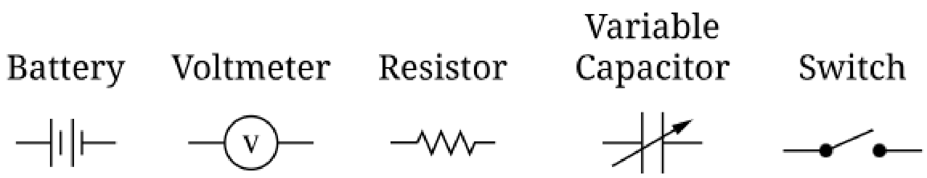
(E) I, II, and III

答案 (B)

Question 18

Read each question carefully. Show all your work for each part of the question. The parts within the question may not have equal weight.

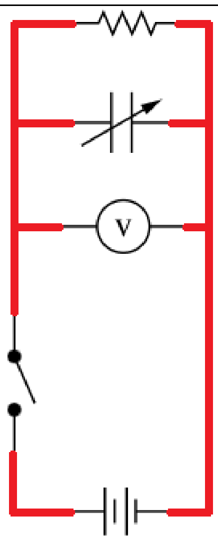
The plates of a certain variable capacitor have an adjustable area. An experiment is performed to study the potential difference across the capacitor as it discharges through a resistor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference ΔV_0, a single voltmeter, a single resistor of resistance R, a single uncharged variable capacitor set to capacitance C, and one or more switches as needed.



(a) Using the symbols shown, draw a schematic diagram of a circuit that can charge the capacitor and may also be used to study the potential difference across the capacitor as it discharges through the resistor.

Setup the circuit as shown in the picture. At time t=0, close the switch so that the battery begins to charge the capacitor. The voltmeter can record the voltage of the capacitor with respect to time.

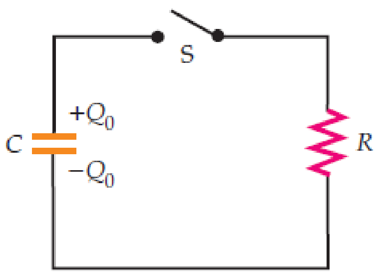
When the capacitor is fully charged, that is to say when we find out that the voltmeter is in a stable voltage, open the switch at time $t=t_1$. The capacitor will discharge through the resistor R. Use the voltmeter to record the voltage of discharging capacitor.



The capacitor is fully charged by the battery. At time t=0, the capacitor starts discharging through the resistor.

(b) Show that the potential difference ΔV_c across the capacitor as a function of time t is $\Delta V_c = \Delta V_0 e^{-\frac{t}{RC}}$ as the capacitor discharges.

At t=0, $I_0 = \frac{V_0}{R} = \frac{Q_0}{RC}$



By the definition of current: $I = -\frac{dQ}{dt}$

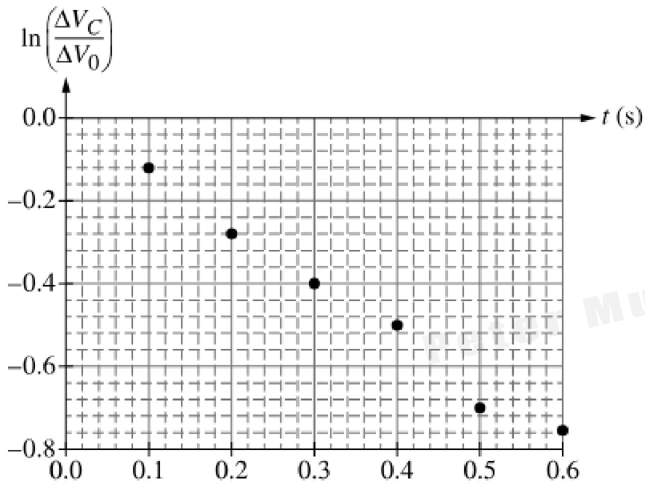
At this time the voltage on the capacitor is equal to the voltage on the resistor, that is:

$\frac{Q}{C} = IR = -R\frac{dQ}{dt} \Rightarrow \frac{dQ}{Q} = \frac{-1}{RC}dt$

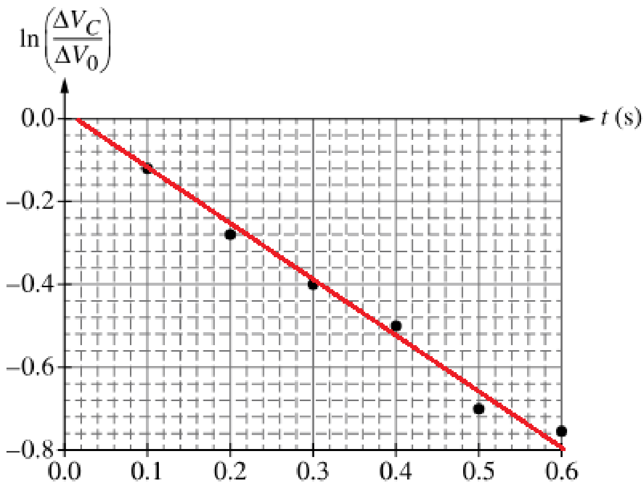
$$\Rightarrow \int_{Q_0}^{Q} \frac{dQ}{Q} = \int_0^t \frac{-1}{RC}dt \Rightarrow \ln\frac{Q}{Q_0} = \frac{-t}{RC} \Rightarrow Q = Q_0 e^{-\frac{t}{RC}}$$

$$\Delta V_c = \frac{Q}{C} = \frac{Q_0}{C}e^{-\frac{t}{RC}} = \Delta V_0 e^{-\frac{t}{RC}}$$

(c) The experiment is performed using a resistor of $R = 150k\Omega$. Data for the potential difference ΔV_c across the capacitor as a function of t are recorded and a plot of $\ln\frac{\Delta V_c}{\Delta V_0}$ as a function of t is created on the graph below.



i. Draw the best-fit line for the data.



ii. Using the best-fit line, calculate a value for the unknown capacitance C.

$$\Delta V_c = \Delta V_0 e^{-\frac{t}{RC}} \Rightarrow \frac{\Delta V_c}{\Delta V_0} = e^{-\frac{t}{RC}} \Rightarrow \ln\frac{\Delta V_c}{\Delta V_0} = -\frac{t}{RC}$$

From the best-fit line, we can get that the slop of the $-\frac{t}{RC}$ is

$$-\frac{t}{RC} = -\frac{0.8}{0.6}t = -\frac{4}{3}t \Rightarrow -\frac{1}{RC} = -\frac{4}{3} \Rightarrow C = \frac{3}{4R} = \frac{3}{4 \times 150k} = 5\ (\mu F)$$

(d) The capacitor is adjusted so that the surface area of the plates is increased, and the experiment is repeated. Would the slope of the best-fit line in the second experiment be steeper, less steep, or unchanged compared to the slope of the best-fit line in part (c)?

___ More steep ___ Less steep ___ Unchanged

Briefly justify your answer.

■ Less steep

As the surface area of the plates is increased, the capacitance is increased accordingly. From $\ln\frac{\Delta V_c}{\Delta V_0} = -\frac{t}{RC}$ we can see that the slope would be less steep.

(e) The ideal battery is then replaced with a non-ideal battery with internal resistance r, and the experiment is repeated.

i. Would the slope of the graph in this final experiment change compared to the graph in part (c)?

___ Yes ___ No

Briefly justify your answer.

■ Yes

With a non-ideal battery with internal resistance r, in the discharge circuit, the equivalent resistance will change from R to R+r. so that the slope of the graph would be less steep.

ii. Would the vertical intercept of the graph in this final experiment change compared to the graph in part (c)?

___ Yes ___ No

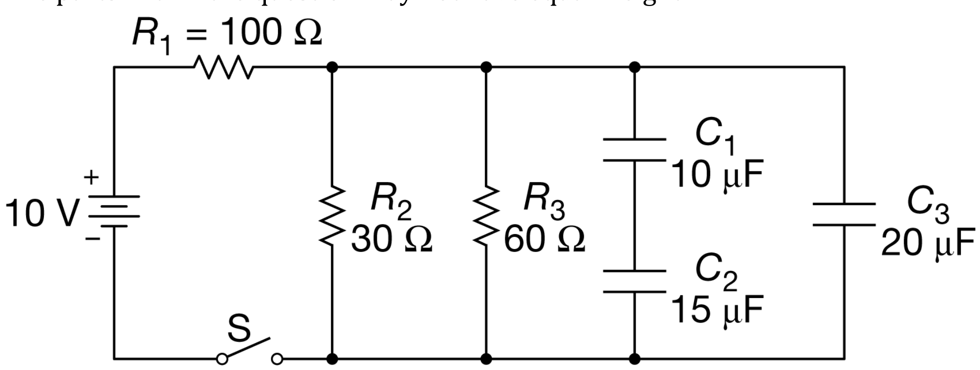
Briefly justify your answer.

■ Yes

With a non-ideal battery with internal resistance r, some voltage would be applied on the internal resistance r, so that the vertical intercept of the graph in this final experiment will change.

Question 19

Read each question carefully. Show all your work for each part of the question. The parts within the question may not have equal weight.



The circuit shown above is composed of an ideal 10V battery, three resistors and three capacitors with the values shown, and an open switch S. The capacitors are initially uncharged. Switch S is now closed.

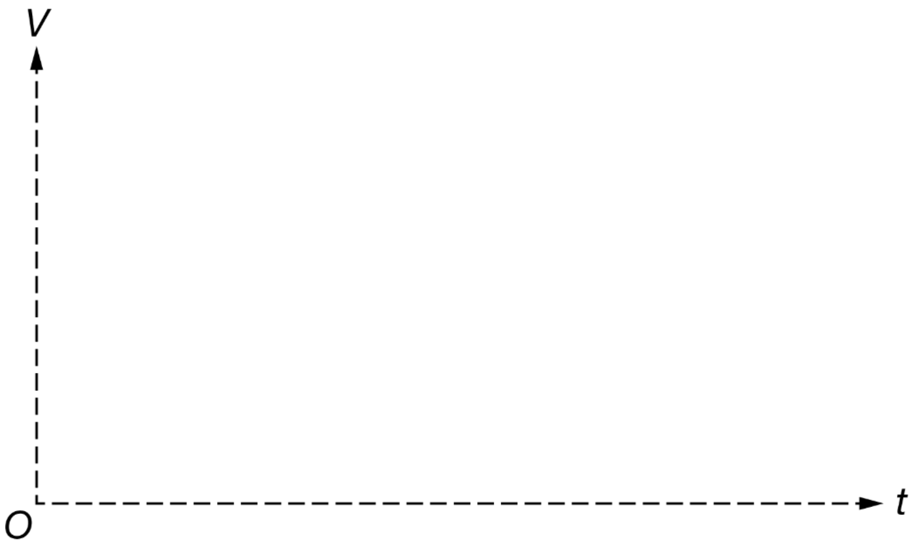
(a) Calculate the current through R_1 immediately after switch S is closed.
Equivalent capacitance of C_1 and C_2: $1/C_{12}=1/10+1/15$, $C_{12}=6$ (μF). Total equivalent capacitance: $C= C_{12}+ C_3=6+20=26$ (μF) For the circuit: $\varepsilon - IR_1 - \frac{Q}{C} = 0$ At t=0, Q=0; so that the equation above is: $\varepsilon - IR_1 = 0 \Rightarrow I = \frac{\varepsilon}{R_1} = I_0 = \frac{10}{100} = 0.1\ (A)$ $I = \frac{dQ}{dt} \Rightarrow \varepsilon - R_1\frac{dQ}{dt} - \frac{Q}{C} = 0 \quad (1)$ Equation (1) is a first-order constant-coefficient non-homogeneous ordinary differential equation, we have: $R_1\frac{dQ}{dt} = \varepsilon - \frac{Q}{C} = \frac{\varepsilon C - Q}{C} \Rightarrow \frac{dQ}{\varepsilon C - Q} = \frac{dt}{R_1 C} \Rightarrow -\frac{d(\varepsilon C - Q)}{\varepsilon C - Q} = \frac{dt}{R_1 C}$ $\Rightarrow -\ln(\varepsilon C - Q)_0^Q = \frac{t}{R_1 C}\Big\|_0^t \Rightarrow -\ln\frac{\varepsilon C - Q}{\varepsilon C} = \frac{t}{R_1 C} \Rightarrow \frac{\varepsilon C - Q}{\varepsilon C} = e^{-\left(\frac{t}{R_1 C}\right)}$ $\Rightarrow \varepsilon C - Q = \varepsilon C e^{-\left(\frac{t}{R_1 C}\right)} \Rightarrow Q = \varepsilon C - \varepsilon C e^{-\left(\frac{t}{R_1 C}\right)} = \varepsilon C\left(1 - e^{\frac{-t}{R_1 C}}\right)$ $\Rightarrow I = \frac{dQ}{dt} = \varepsilon C\left(-e^{\frac{-1}{R_1 C}} e^{\frac{-t}{RC}}\right) = \frac{\varepsilon}{R_1} e^{\frac{-t}{R_1 C}} = I_0 e^{\frac{-t}{R_1 C}}$
Switch S has been closed for a long time, and the circuit has reached a steady state. (b) Calculate the potential difference across R_1.
Equivalent resistance of R_2 and R_3: $1/R_{23}=1/30+1/60 \Rightarrow R_{23}=20\Omega$. Current on R_1: $I_1=V/(R_1+R_{23})=10/(100+20)=1/12$ (A)
(c) i. Calculate the charge stored on the positive plate of capacitor C_2.
Equivalent capacitance of C1 and C2: $1/C_{12}=1/10+1/15$, $C_{12}=6$ (μF). Total equivalent capacitance: $C= C_{12}+ C_3=6+20=26$ (μF) Voltage across R_2 is the voltage across capacitor C_1+C_2: $V_{23}=I_1{*}R_{23}=(1/12)(20)=5/3$ (V) $\quad V_{23}=V_{C1C2}=5/3$ (V) Charge on the positive plate of capacitor C_2 equals to the Charge on the positive plate of capacitor C_1: $V=Q/C \Rightarrow Q_{12}= V_{C1C2}\, C_{12}=(5/3)(6)= 10$ (μC) It is connected with the anode of the battery so that the charge is positive. The charge is +10μC
ii. Is the charge stored on capacitor C_3 greater than, less than, or equal to the charge stored on capacitor C_2 ? ____ Greater than ____ Less than ____ Equal to Justify your answer.
$V=Q/C \Rightarrow Q_3= V_{C1C2}C_3=(5/3)(20)=100/3$ (μC) $\Rightarrow Q_3> Q_{12}= Q_2$ _■_ Greater than
Switch S is then opened. (d) i. Determine the current through R_1 immediately after the switch is opened.

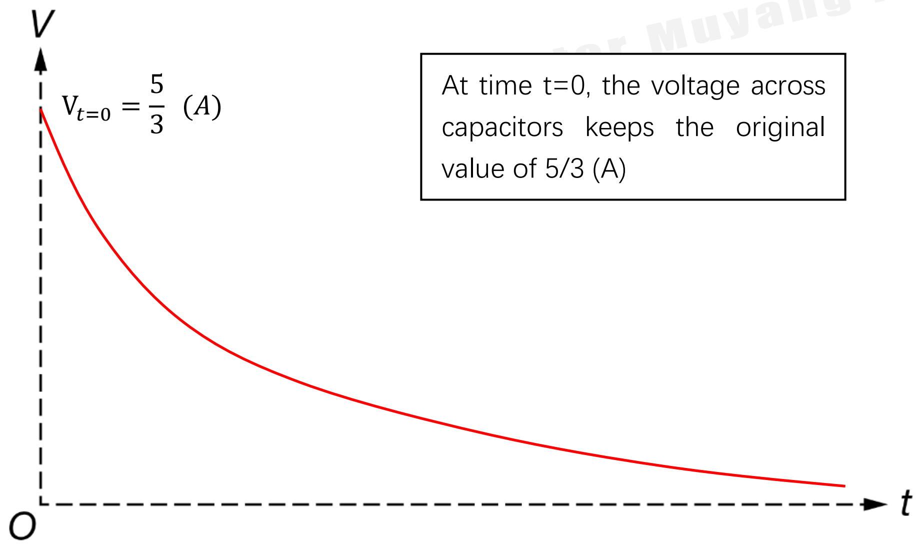
After S is opened, the R_1 is in an open circuit so that the current in it is zero.

ii. Calculate the current through R_2 immediately after the switch is opened.

After switch S is opened, capacitors serve as a battery to charge R_2 and R_3:
Voltage immediately after the switch is opened across R_2(or R_3): $V_{23}=5/3$ (V)
Current in R_2: $I_2R_2=V_{23}=Q/C$
$I_2= V_{23}/ R_2=(5/3)/30=5/90$ (A)

(e) On the axes below, sketch a graph of the potential difference V across capacitor C_2 as a function of time t if switch S is opened at time t=0. Label the maximum value.



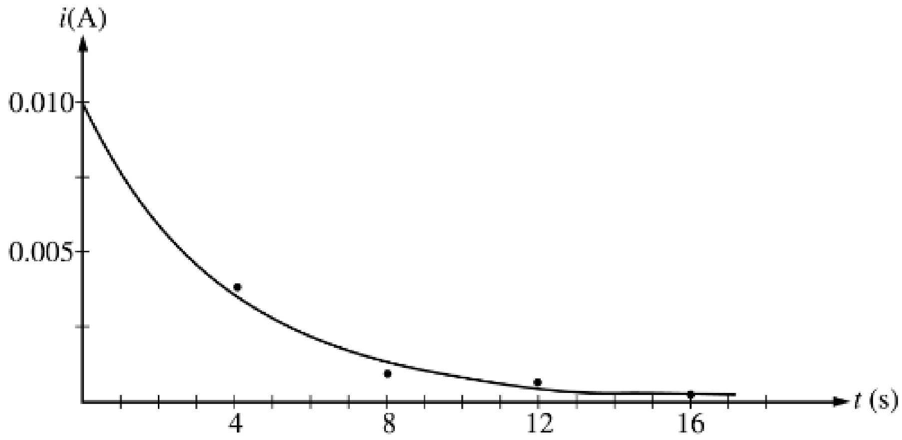


Capacitor C_3 is replaced by two 10μF capacitors connected in series, switch S is closed, and the circuit reaches equilibrium. Switch S is then opened at time t=0.
(f) For t>0, would the sketch of a graph of the new voltage across C_2 as a function of time be above, below, or the same as the sketch for part (e)?
____ Above ____ Below ____ The same
Justify your answer.

The equivalent capacitance of two 10μF capacitors connected in series is 5μF<C_3=20μF, so that the total capacitance in the new circuit is: 5+6=11 (μF).
In the new circuit, the capacitance is smaller so that it can store less charge.
Because of this issue, the new curve will be below the original one.
__■__ Below

Question 20

In the laboratory, you connect a resistor and a capacitor with unknown values in series with a battery of emf $\mathcal{E}$ = 12 V . You include a switch in the circuit. When the switch is closed at time t = 0, the circuit is completed, and you measure the current through the resistor as a function of time as plotted below.



A data-fitting program finds that the current decays according to the equation

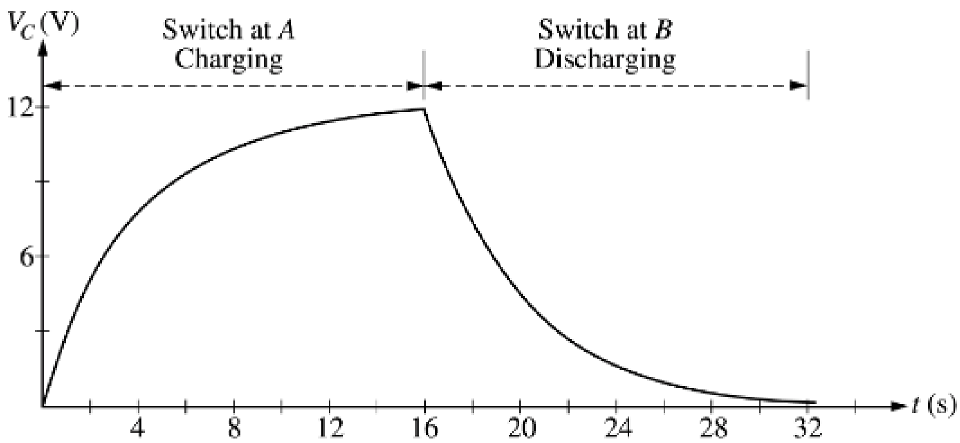
$i(t) = \frac{\mathcal{E}}{R} e^{\frac{-t}{4}}$.

a. Using common symbols for the battery, the resistor, the capacitor, and the switch, draw the circuit that you constructed. Show the circuit before the switch is closed and include whatever other devices you need to measure the current through the resistor to obtain the above plot. Label each component in your diagram.

b. Having obtained the curve shown above, determine the value of the resistor that you placed in this circuit.

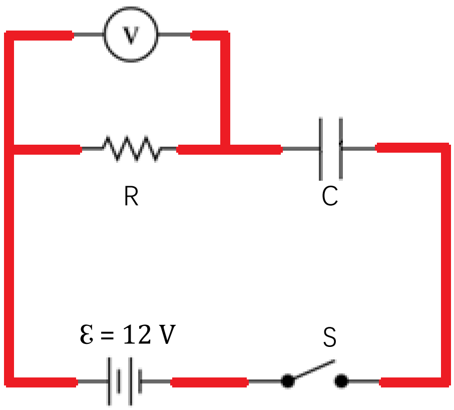
c. What capacitance did you insert in the circuit to give the result above?

You are now asked to reconnect the circuit with a new switch in such a way as to charge and discharge the capacitor. When the switch in the circuit is in position A, the capacitor is charging; and when the switch is in position B, the capacitor is discharging, as represented by the graph below of voltage V_C across the capacitor as a function of time.



d. Draw a schematic diagram of the RC circuit that you constructed that would produce the graph above. Clearly indicate switch positions A and B on your circuit diagram and include whatever other devices you need to measure the voltage across the capacitor to obtain the above plot. Label each component in your diagram.

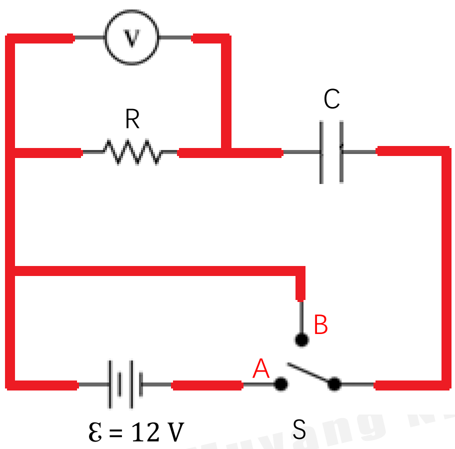
a. A circuit is setup in the right figure. Use a voltmeter to measure the voltage across the resister, so that with the equation I=V/R, we can obtain the current of the resister. As the resister are in series with the capacitor, same current applied in the capacitor.



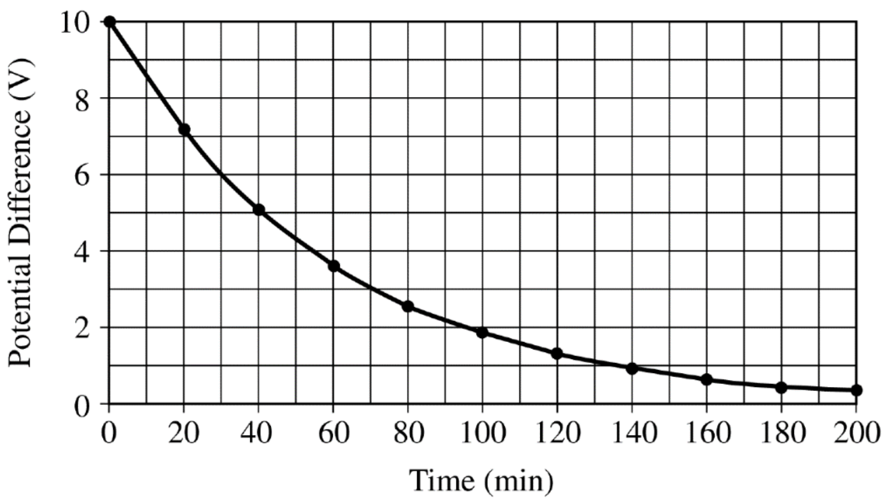
b. $I_0 = \frac{\mathcal{E}}{\mathrm{R}} \Rightarrow R = \mathcal{E}I_0 = 12 \times 0.01 = 0.12\ (\Omega)$

c. $i(t) = \frac{\mathcal{E}}{\mathrm{R}} e^{\frac{-t}{4}} = \frac{\mathcal{E}}{\mathrm{R}} e^{\frac{-t}{RC}} \ \Rightarrow RC = 4 \ \Rightarrow C = \frac{4}{0.12} = 33\ (F)$

d. When the switch is in position A, a EMF of Ɛ = 12 V is connected in the circuit and it charges the capacitor C in the circuit. When the switch alters to position B, the capacitor will discharge through resistor R.



Question 21



You have been hired to determine the internal resistance of 8.0 μF capacitors for an electronic component manufacturer. (Ideal capacitors have an infinite internal resistance—that is, the material between their plates is a perfect insulator. In practice, however, the material has a very small, but nonzero, conductivity.) You cannot simply connect the capacitors to an ohmmeter, because their resistance is too large for an ohmmeter to measure. Therefore you charge the capacitor to a potential difference of 10 V with a battery, disconnect it from the battery and measure the potential difference across the capacitor every 20 minutes with an ideal voltmeter, obtaining the graph shown above.

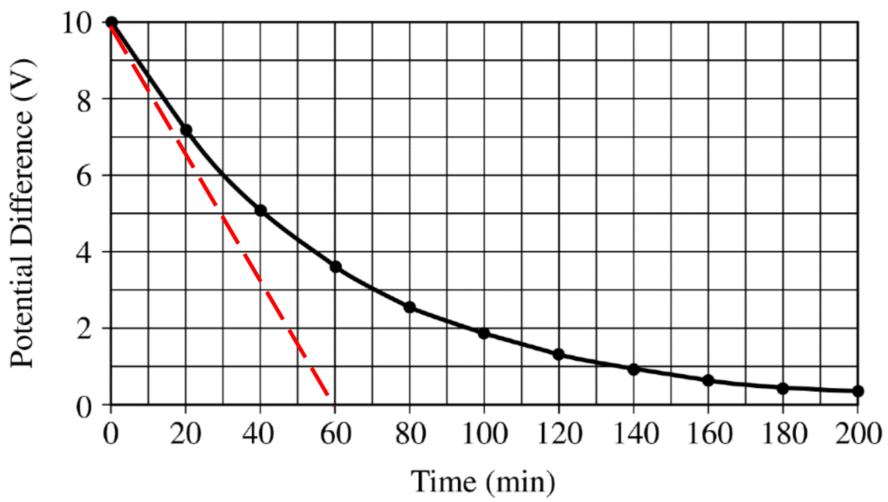
a. Determine the internal resistance of the capacitor.

The capacitor can be approximated as a parallel-plate capacitor separated by a 0.10 mm thick dielectric with $\kappa = 5.6$.

b. Determine the approximate surface area of one of the capacitor "plates."

c. Determine the resistivity of the dielectric.

d. Determine the magnitude of the charge leaving the positive plate of the capacitor in the first 100 min.



a.

RC=60 (min)= 3600 (s) ⇒ R=3600/(8*10^{-6})=450 (MΩ)

b. $C = \frac{\kappa\varepsilon_0 A}{d} \Rightarrow A = \frac{Cd}{\kappa\varepsilon_0} = \frac{(8\times10^{-6})(0.10\times10^{-3})}{(5.6)(8.85\times10^{-12})} = 16\ (\mathrm{m}^2)$

c. $R = \rho\frac{L}{A} \Rightarrow \rho = \frac{RA}{L} = \frac{(450\times10^{6})(16)}{0.10\times10^{-3}} = 7.2\times10^{13}\ (\Omega\cdot\mathrm{m})$

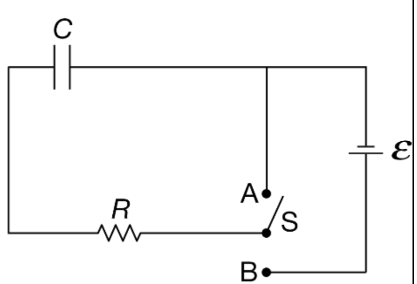
d. At t=100 (min), V=2 (V)

$V = \frac{Q}{C} \Rightarrow Q = VC = 2\times(8\times10^{-6}) = 16\times10^{-6}\ (C) = 16\ (\mu C)$

Question 22

Read each part carefully. Show all your work for each part of the question. The parts within the question may not have equal weight.

A group of students are to experimentally determine the capacitance C of capacitor in a circuit as shown. The other components of the circuit are a resistor with resistance R, a battery with potential difference Ɛ and switch S. In addition, there are several different resistors that are available for the students to use. The students are expected to collect data as the capacitor is discharging and then find the capacitance through graphical analysis. The students have access to equipment typically found in a high school physics lab but must use a setup similar to the one shown above.

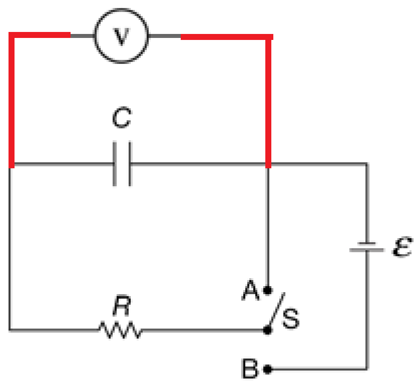


(a) Clearly identify each quantity to be measured, the symbol used to represent that quantity, and the equipment that would be used to measure the quantity.

1. The resistance R in Ω of the resister.
2. The voltage of the capacitor V(t) with respect to time in Volt.

(b) Describe the procedure a student would use to collect data that would allow the student to determine the capacitance C of the capacitor. Provide enough detail so that another student could replicate the experiment, including any steps necessary to reduce experimental uncertainty. As needed, use the symbols defined in part (a) above.

1. Dis-connect the resister from the circuit. Use an ohmmeter to measure the resistance R in Ω of the resister.
2. Setup the circuit shown in the right figure. Use the voltmeter to record the voltage of the capacitor in V.
3. Turn the switch to position B so that the capacitor can be fully charged by the battery. At time t=0, alter the switch to position A and record the voltage V(t) with respect to time.



(c) Which quantities (raw data or calculated from the data) would be graphed on the horizontal and vertical axes to produce a graph that could be used to determine the capacitance C of the capacitor?

The horizontal axis should be time in seconds.
The vertical axis should be voltage of the capacitor in volts.

(d) Describe the process by which the students would use the graph described in part (c) to determine the capacitance C of the capacitor. Include the relevant feature of the graph that would be used and how it would be used to determine the capacitance C of the capacitor.

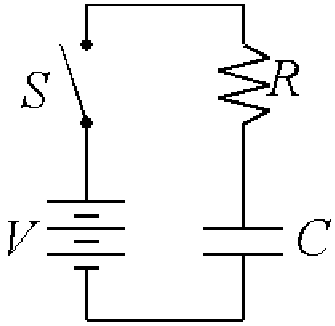
The student should get a curve plotted in V-t as described in part (c)
find the tangent line of the curve at time t=0 and it intersect with the x-axis at time t.
$t=RC \Rightarrow C=t/R$. Use the measurement result of R and the time t to calculate the capacitance C.

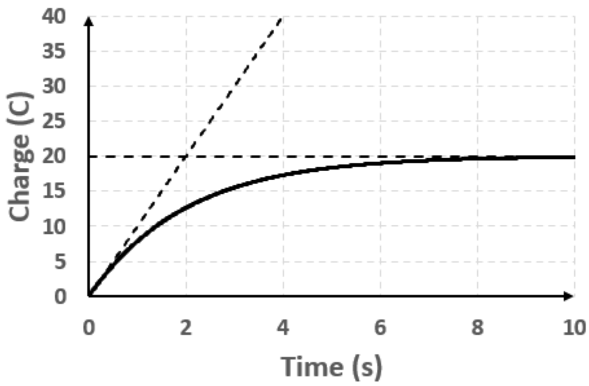
(e) The students determine a value for C that is much less than the capacitor's advertised capacitance, and they suspect that there is an error in their equipment or experimental design. Explain one experimental factor that could account for the students' determining a value for the capacitance that is much less than the actual value, and justify your answer.

If the voltmeter's internal resistance is wrong, that is to say, the actual equivalent resistance is smaller, so that the calculation based on this situation would generate a smaller competence of C.

Question 23

This question should take approximately 25 minutes. Read each of the parts carefully. Support your answers with relevant physics principles. The parts within the question may not have equal weight. Write your answers in the parts provided after each part. If you find you are in need of symbolic notation that is too difficult to type, you may identify a non-standard symbol and use that throughout your answer. For example, you may identify "Q" as the measurement for an angle.





A student connects a battery with constant potential difference ΔV, a switch S, a resistor R, and a capacitor C as shown in the figure. At time t=0, the switch is closed. The graph shows the net charge that has passed through the battery starting at time t=0. Positive values represent positive charge leaving the positive terminal. There are also dashed lines in the graph representing the line tangent to the graph at time t=0 and the line that the graph asymptotically reaches after a long time.

(a) Suggest values for V in volts, R in ohms, and C in farads that could result in the graph shown above. Explain your choice of values.

At time t=0, Q=0 (C)

At time t=∞, Q= 20 (C)

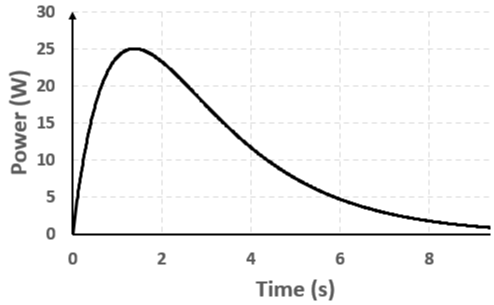
The tangent line at t=0 intersect the Q=20 at a time of t=2 (s)

(b) A student is asked to create a second graph with twice the initial slope but the same horizontal asymptote as the original graph. The student has access to one additional battery ΔV, one additional resistor R, and one additional capacitor C, along with several additional connecting wires. Explain how one or more of the additional components could be added to the circuit to accomplish this task. Justify your answer.

The charge Q(t) will satisfy: $Q(t) = Q_0(1 - e^{\frac{-t}{RC}})$ where $Q_0 = \frac{\Delta V}{C}$

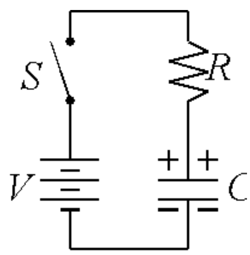
1. With the additional battery ΔV, Q_0 will double. That is to say the slop of the curve will be twice of the initial one.
2. A second method is to add a resister in parallel with the original one so that the total resistance would be half of the original one, which will lead to twice of the slop.
3. A third method is to add a capacitor in serial with the original one. It also introduces a new value of capacitance of half of the original one and also leads to twice of the slop.

(c) The student constructs the above graph of the power delivered to the capacitor as a function of time. Briefly explain why the graph starts at zero, reaches a maximum, and then asymptotically approaches zero again.

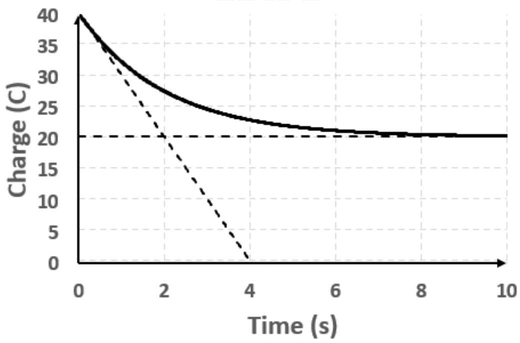


At time t=0, the voltage across the capacitor V(0)=0 and the current of it is I(0)=V/R. That is to say the current is the maximum value.
The Power P=IV so that P(0)=0.
As time pass by, the voltage will increase gradually until it achieved the maximum value of V(t)=V while the current of it will decrease accordingly until it reaches I(∞)=0.
The Power P=IV so that P(∞)=0.
At some time between t=0 and t=∞, the power can reach its maximum value so that P(t)>0=P_{max}.

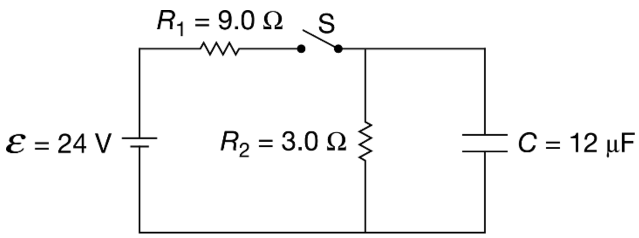
(d) The student repeats the experiment, however this time the capacitor is initially charged so that its top plate is positive and the potential difference between the plates is twice the potential difference across the battery. Describe how the graph of charge vs. time from part (c) would appear now if the switch is closed at time t=0. Be sure to address initial slope and final asymptote of the graph. Justify your answers.



As the potential difference between the plates is twice the potential difference across the battery at time t=0, the potential of across the resistor is 2V-V=V.
So that the capacitor will discharge until the volte reaches V.
The right figure shows the discharging procedure of Charge Q with respect to time t.



Question 24
Read each question carefully. Show all your work for each part of the question. The parts within the question may not have equal weight.



A power supply is set to Ɛ =24V and is connected to resistors R_1=9.0Ω and R_2=3.0Ω , capacitor C=12μF , and switch S , as shown in the figure. Initially, the capacitor is uncharged, and switch S is open.

(a) At time t=0 , the switch is then closed.
i. Calculate the current through R_1 immediately after the switch is closed.

After the switch is closed, the capacitor branch is short.
The current through R_1 is I_1 : $I_1= \mathcal{E}/ R_1 =24/9=8/3$ (A)

ii. Determine the current through R_2 immediately after the switch is closed.

As the capacitor branch is short, the voltage across C and R_2 is equal 0.
The current through R_2 is I_2 :$I_2=0/3.0=0$ (A)

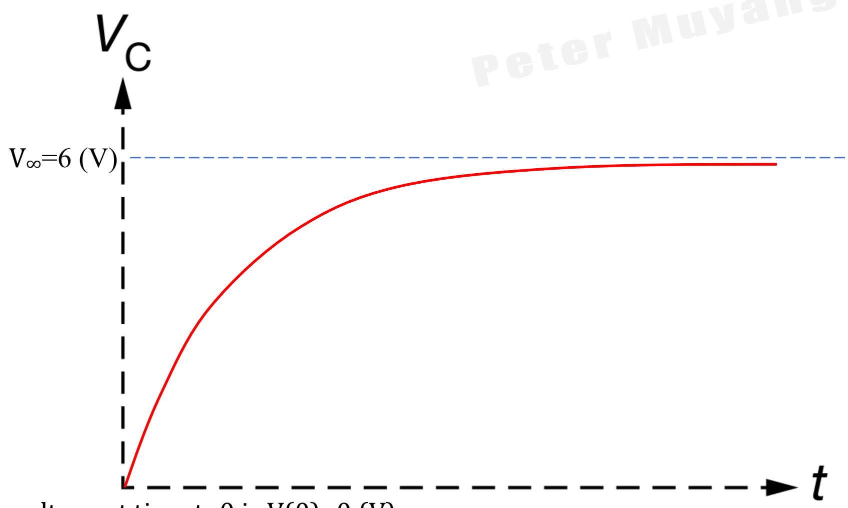
A long time after the switch is closed, the circuit reaches steady-state conditions.
(b) Calculate the potential difference across R_2 .

A long time after the switch is closed, the circuit reaches steady-state conditions so that the current through C is 0 and the voltage across C equals to the potential difference across R_2.
$R_{12}=R_1+R_2=12$ (Ω)
The voltage across R_2 is V_2 : $V_2/ \mathcal{E}= R_2/ R_{12}$ $V_2=6$ (V)

(c) Calculate the magnitude of the charge Q on the positive plate of the capacitor.

Q=V*C=6*12=72 (μC)

(d) On the axes shown, sketch a graph of the potential difference V_C across the capacitor as a function of time t . Explicitly label any intercepts, asymptotes, maxima, or minima with values or expressions, as appropriate.



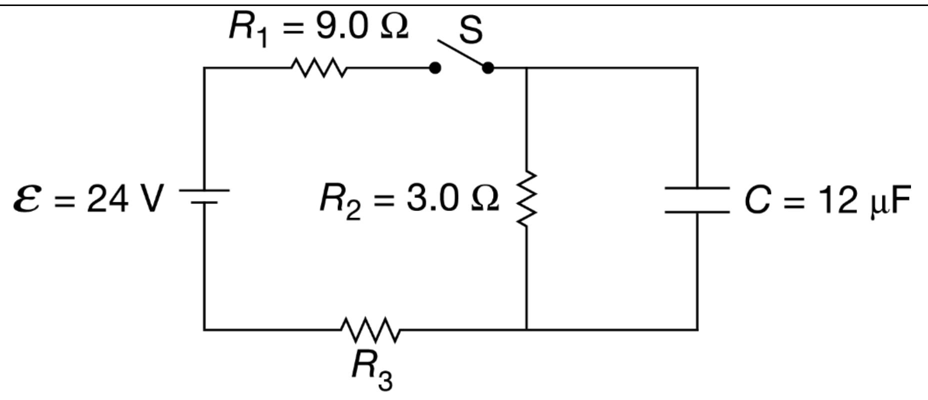
The voltage at time t=0 is V(0)=0 (V)
The voltage at time t=∞ is V(∞)=6 (V)

After steady-state conditions are reached, the switch is now opened, and time is reset to t=0.

(e) Using integral calculus, derive an expression for the charge q(t) on the capacitor as a function of time t after the switch is opened. Express your answer in terms of Q.

After the switch is opened, the capacitor will discharge through resister R_2.

$q(t) = Q_0 e^{\frac{-t}{RC}}$ where $Q_0 = 72\ \mu C$, C=12 μF, R= R_2=3.0 Ω.



The capacitor is discharged, and a third resistor is added to the circuit, as shown above. The switch is then closed

(f) Does the time it takes for the charge on the capacitor to reach 2/3 of its maximum value increase, decrease, or stay the same as compared to the circuit in part (a)?

_____Increase _____Decrease _____Stay the same

Justify your answer.

As total resistance is increased after the new resistor is inserted, according to the charging equation:

$$Q(t) = Q_0 e^{\frac{-t}{RC}}$$

Where R is increased. So that the time it takes for the charge on the capacitor to reach 2/3 of its maximum value would increase.

__■__Increase